Unit Two

2.8

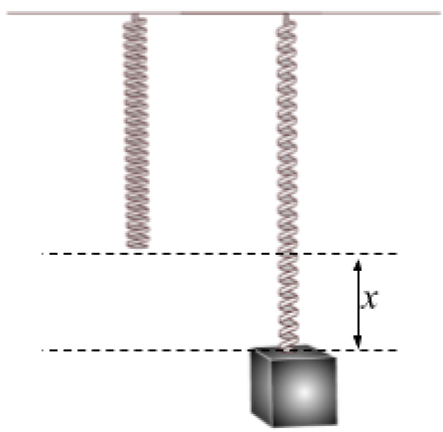
Spring Forces

2.8 Spring Forces

Learning Objectives:-

- Describe the force exerted on an object by an ideal spring.
- Describe the equivalent spring constant of a combination of springs exerting forces on an object.

Forces Exerted by Spring (Stretched or Composed)



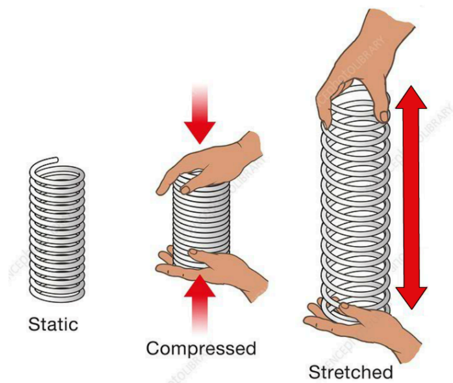
A force must be applied to displace the spring from the system of equilibrium position.
An internal force is exerted when the spring is displaced directed back toward the system's equilibrium position (Restoring force).
The ratio between the restoring force and displacement is known as the spring constant (K).

- *Spring constant (k) is how much force is needed to apply to stretch or compress the spring by 1 m.*

If the relation between the applied force and displacement is linear, then the spring is considered an ideal sporing.

Equilibrium Position:

- The equilibrium position of a spring is where the spring naturally rests without any external forces acting on it.
- To **displace** the spring from this position (stretch or compress it), a **force** must be applied.

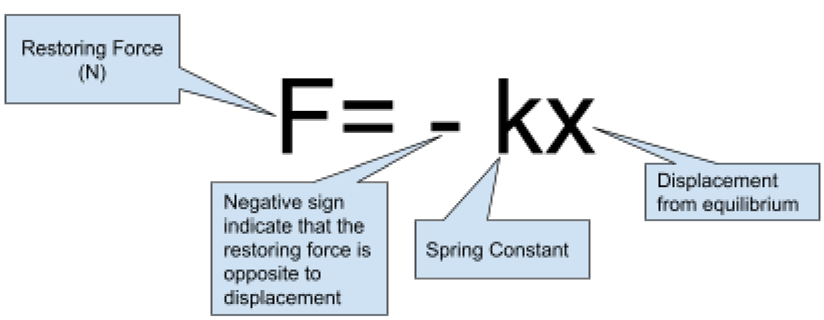


Restoring Force:

- When a spring is displaced from its equilibrium position, it generates an **internal force** that resists this displacement.
- This **restoring force** always acts in the opposite direction of the displacement and tries to return the spring to its equilibrium position.

Hooke's Law:

- The relationship between the restoring force (F) and the displacement (x) is described by



- F *is the restoring force in Newtons* (N)
- K *is the spring constant* (N/m)
- x *is the displacement from equilibrium position* (m).

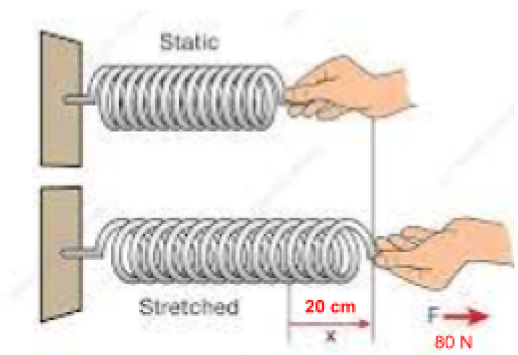
Spring Constant (k):

- The spring constant measures the stiffness of a spring, and tells how much force is required to stretch or compress the spring by 1m.

Linear Relationship:

- If the relationship between the applied force and the displacement of the spring is linear, the spring obeys Hooke's Law.
- This linear behavior occurs only within the spring's elastic limit (the range where it doesn't deform permanently).

Example:- A force of 80 N applied to an ideal spring to stretch it 20 cm. How much force would need to be applied to compress the spring 8.5 cm?



ANSWER:-

Givings:-

$F = 80\,N$
$x = 20\,cm$

Formula

$F = -kx$

Solution

First we find the spring constant (k).

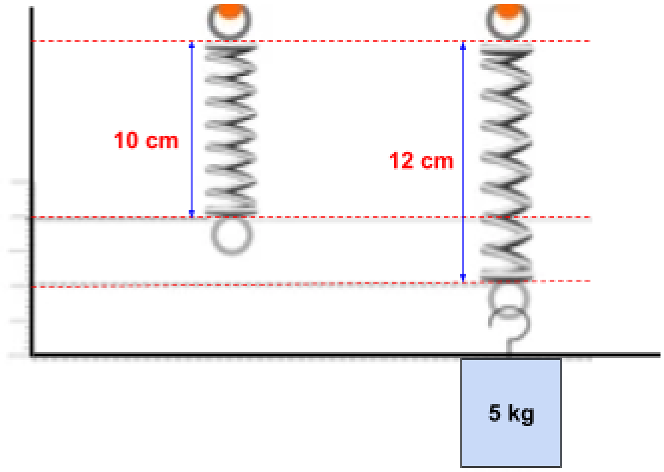
$80\,N = -k(20\,cm)$

$k = \frac{80\,N}{20\,cm} = 4\,N/cm.$

So to compress the spring by 8.5 cm, we need to apply force of

$F = -kx \Rightarrow F = -4 \times 8.5 = 34\,N$

Example (2):- depending on the graph, find the spring constant (K).



ANSWER:-

Givings:-

$m = 5\,kg$
$x = 12\,cm - 10\,cm = 2\,cm$
$2\,cm = \frac{2}{100} = 0.02\,m$

Formula

$F = -kx$

Solution

The applied force is the weight of the mass
$F = mg = 5 \times 10 = 50\,N$
$50N = -k(0.02m)$
$k = \frac{50\,N}{0.02\,m} = 2,500\ N/m.$

Multiple springs (Parallel and Series)

1. **Connecting springs in series**

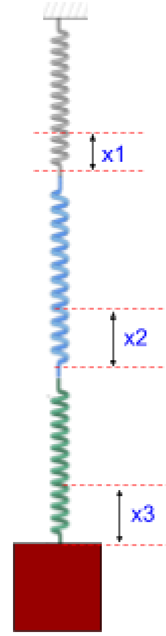
When connecting springs one after another (series), the total displacement equals the sum of individual springs displacement.

$x = x_1 + x_2 + x_3 + ...$

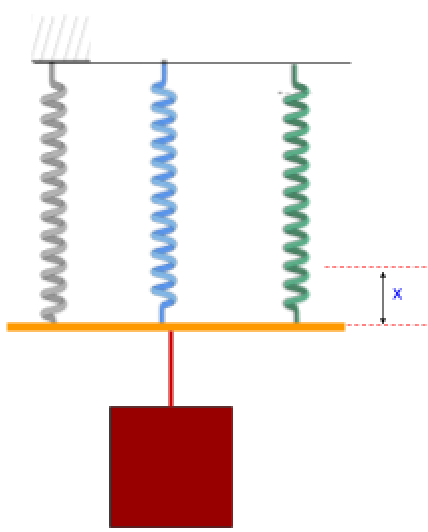
Total restoring force is the combination of springs restoring forces, which equals to restoring force exerted by each spring.

When springs in series, the equivalent spring constant can be found through

$$\frac{1}{k_{eq}} = \frac{1}{k_1} + \frac{1}{k_2} + \frac{1}{k_3}$$



2. **Connecting springs in parallel**



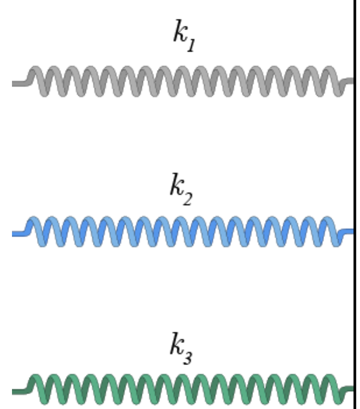
When connecting springs in parallel, the total displacement equals the displacement of the individual spring.

The restoring force equals force exerted by each spring added together.

$$K_{eq} = k_1 + k_2 + k_3$$

Example:- three springs each has different spring constant
$k_1 = 100\, N/m,\ k_2 = 150\, N/m,\ K_3 = 50\, N/m$

a. Determine the equivalent spring constant if these springs are connected in series.
b. What is the equivalent spring constant if they are connected in parallel?
c. Determine how far each system would displace under a force of 100 N?



ANSWER:

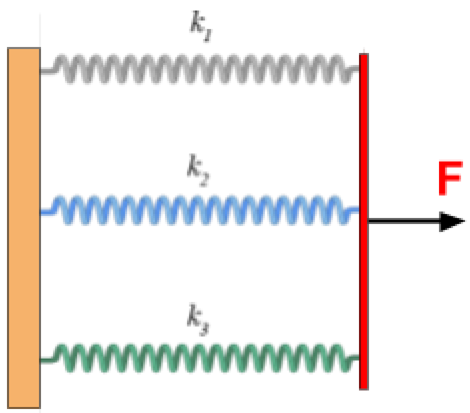
a. **Connecting springs in series**

$$\frac{1}{k_{eq}} = \frac{1}{k_1} + \frac{1}{k_2} + \frac{1}{k_3} \Rightarrow$$

$$\frac{1}{k_{eq}} = \frac{1}{100} + \frac{1}{150} + \frac{1}{50} \Rightarrow \frac{1}{k_{eq}} = \frac{11}{300}$$

$$k_{eq} = \frac{300}{11} = 27.3\, N/m$$

b. Connecting springs in Parallel



$$k_{eq} = k_1 + k_2 + k_3$$

$$k_{eq} = 100 + 150 + 50 = 300\, N/m$$

c. Displacement

- For the springs in series

$$F = -kx \Rightarrow x = \frac{F}{k} = \frac{100}{27.3} = 3.66m$$

- For the springs in Parallel

$$F = -kx \Rightarrow x = \frac{F}{k} = \frac{100}{300} = 0.33\, m$$

Worksheet

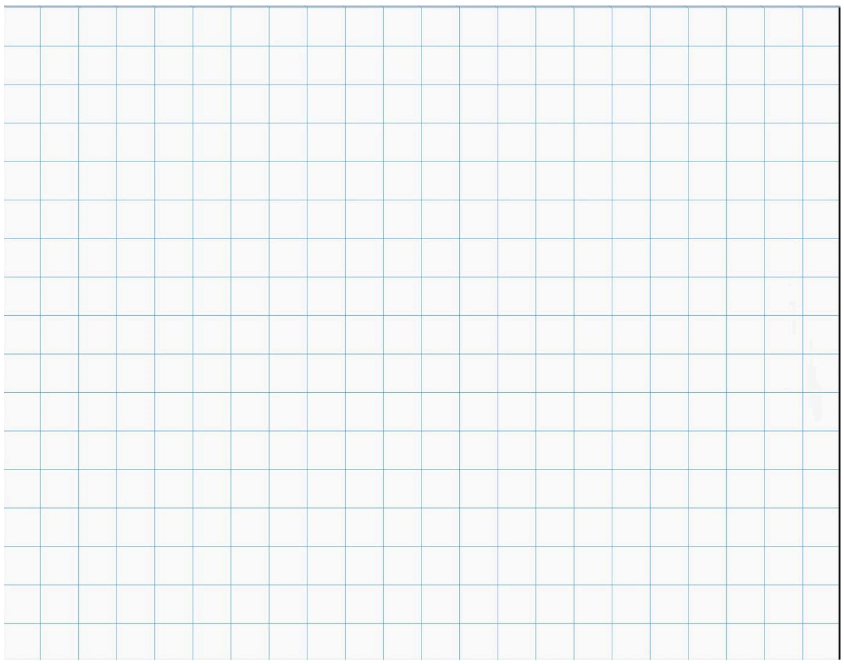
2.8

Spring forces

Q1: A spring is hung vertically from support and various measles are attached to the spring as shown in the table

Weight (N)	0	30	35	40	50	60
Length (m)	0.20	0.4	0.43	0.47	0.53	0.6

a. Represent the data in a graph, and use it to find the spring constant.



b. If an unknown mass is hung from the spring and the new length of the spring is measured to be 0.9 m. determine the mass.

Q2: A mass of 2 kg attached to the vertical spring causing it to stretch. The length of the spring after attaching the mass is 12 cm, and when the 2kg mass is replaced by 5 kg mass, the new spring length is 15 cm. Assume that the spring follows Hooke's law. What is the spring constant (k)?